

# The $R_{uds}$ value in the vicinity of $\psi(3770)$ state

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## Abstract

The anomalous line shape of the  $\psi(3770)$  state has resulted in some difficulty in the determination of the  $R$  value for the continuum light hadron production in the resonance energy range. We parameterize the asymmetric line shape using a Fano-type formula and extract the  $R_{uds}$  value to be  $2.156 \pm 0.022$  from the data of BESIII Collaboration in the energy region between 3.650 and 3.872 GeV. The small discrepancy between experiment and theory is removed. The cross sections of the  $e^+e^- \rightarrow \text{hadrons}$  are given by subtracting the part of continuum light hadron production and are compared to the data of the  $e^+e^- \rightarrow D\bar{D}$  reaction.

**Keywords:** R value, Fano resonance,  $\psi(3770)$

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## 1. Introduction

The cross section of the  $e^+e^- \rightarrow \text{hadrons}$  in terms of the center-of-mass (c.m.) energy is one of the most fundamental observables in Quantum Chromodynamics (QCD). The final hadrons are produced via a pair of quark-antiquark proceeded from a virtual photon by initial-state electron-positron annihilation. Instead of the cross section for inclusive hadron production, the hadronic R-ratio  $R(s)$  is often used owing to its simplicity both on the experimental and the theoretical side,

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}, \quad (1)$$

where  $\sigma(e^+e^- \rightarrow \mu^+\mu^-) = 4\pi\alpha^2/3s$  is the photon-mediated lowest order muon pair production cross section with  $s$  being the square of c.m. energy and  $\alpha$  the electromagnetic coupling constant. If no resonances are present, the  $R(s)$  values solely from the continuum hadrons are well known to be  $3 \sum_f Q_f^2$  in the lowest order approximation, with  $f$  being quark flavors and  $Q_f$  the corresponding quark charge. The higher order corrections from the finite quark masses and the gluonic emission could be calculated by perturbative QCD (pQCD) [1, 2, 3]. So the measurement of  $R(s)$  is important for testing the validity of both pQCD calculation and hadronic vacuum polarization correction.

The  $R(s)$  for the continuum light hadron (containing  $u$ ,  $d$  and  $s$  quarks) production, denoted as  $R_{uds}$  in this letter, is usually used to test the validity of the pQCD calculation in relatively low energy region. Precise measurements of the  $R_{uds}$  near the  $D\bar{D}$  threshold are reported by BES Collaboration [4, 5, 6, 7]. The  $R_{uds}$  value below the  $D\bar{D}$  threshold is not affected by the first  $D\bar{D}$  open charm resonance  $\psi(3770)$ , therefore, determination of  $R_{uds}$  is very simple for the case. The  $R_{uds}$  in the energy region from 3.650 to 3.732 GeV is determined to be  $R_{uds} = 2.141 \pm 0.025 \pm 0.085$  [4], which is in good agreement with  $R_{uds}^{pQCD} = 2.15$  predicted by pQCD [1, 2, 3]. However, the  $R_{uds}$  value in the open charm threshold region is overlapped with many resonances. The obtained value varies widely among different fits, e.g. in or nearby the  $\psi(3770)$  resonance. It is extracted to be  $R_{uds} = 2.262 \pm 0.054 \pm 0.109$  in the energy region from 3.660 to 3.872 GeV [5] and to be  $R_{uds} = 2.121 \pm 0.023 \pm 0.084$  in the energy region from 3.650 to 3.872 GeV [6]. There are obvious differences among the obtained  $R_{uds}$  values and also the pQCD calculation. As a matter of fact,

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these extracted  $R_{uds}$  values depend on the treatment of resonances in the  $\psi(3770)$  region. Therefore, a more reliable method is required to reasonably extract the  $R_{uds}$  value from the experimental data.

In order to accurately extract the  $R_{uds}$  in the region of  $\psi(3770)$  resonance, the anomalous line shape of the  $\psi(3770)$  state should be treated carefully. It has been found at the very beginning that the total cross sections of  $e^+e^- \rightarrow \text{hadrons}$  in the energy range between 3.700 and 3.872 GeV could not be described well with only one Breit-Wigner (BW) resonance even using the energy-dependent width of the  $\psi(3770)$  [4, 5, 6, 7]. This is confirmed by the inclusive measurements of the  $e^+e^- \rightarrow D\bar{D}$ ,  $D^+D^-$ ,  $D^0\bar{D}^0$  reactions in the similar c.m. energy region [8, 9]. In the analysis of the BES Collaboration, a modified form of the energy-dependent width is usually used in their fits to data,

$$\Gamma_{D\bar{D}}(s) \propto \frac{p_{0,\pm}^3(s)}{1 + r^2 p_{0,\pm}^2(s)}, \quad (2)$$

with  $p_{0,\pm} = \sqrt{s/4 - m_{D^{0,\pm}}^2}$  being the final  $D$ -meson momentum in the c.m. system and  $r \sim 1.0$  fm the interaction radius of the  $c\bar{c}$ . The BW resonance with the width in Eq. (2) could give an asymmetric line shape of the  $\psi(3770)$  state, but does not describe well the dip around 3.82 GeV.

The deviation from the BW resonance of the  $\psi(3770)$  state has inspired a lot of interesting theoretical efforts [10, 11, 12, 13, 14, 15, 16, 17, 18]. Experimental measurements from both BES and CLEO Collaborations show that the  $\psi(3770)$  resonance mainly decays to  $D\bar{D}$  channel though the specific decay ratio is still under discussion [19, 20, 21, 22, 23, 24]. However, the rescattering of final  $D\bar{D}$  is found to be not enough to account for its line shape deviation [10]. Now it is explained to be the consequence of the interference between the  $\psi(3770)$  resonance and the continuum background from the  $\psi(2S)$  contribution [11, 12, 13, 14, 15, 16]. Its implication to the nature of  $\psi(3770)$  state is also investigated in the Fano mechanism [17, 18]. In the Fano theory, the asymmetric line shape of states is produced by the interference of continuum and resonance, which is giving rise to a general physical phenomenon in many quantum system, e.g. the nuclear, atomic, condensed matter physics and molecular spectroscopy. Though the underlying physics of the  $\psi(3770)$  state is still waiting for further exploration [17, 18], the Fano-type formula provides an appropriate and simple parameterized expression for describing the anomalous line shape of the total cross sections of the  $e^+e^- \rightarrow \text{hadrons}$  and  $D\bar{D}$ . In this letter, we will use this formula to extract the  $R_{uds}$  value from experimental data reported by BES Collaboration in the energy region between 3.650 and 3.872 GeV[4].

## 2. Method and Result

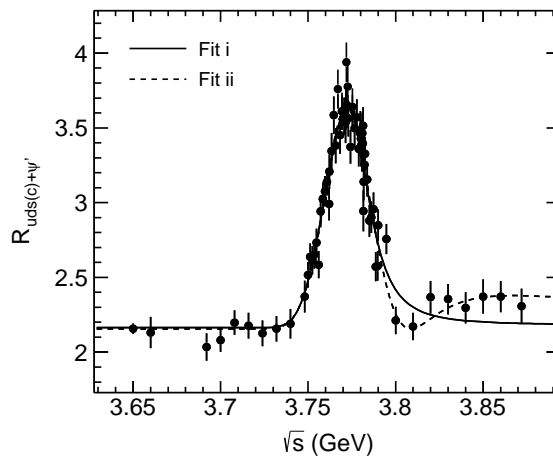


Figure 1:  $R_{uds(c)+\psi'}(s)$  at different c.m. energies. The curves are the fits to the data with (solid line) and without (dashed line)  $\Gamma_{\psi'e^+e^-}$  fixed to the average value in PDG. The data are measured by BESIII Collaboration [4].

The theoretical  $R_{uds(c)+\psi'}(s)$  contains the contributions from continuum light hadron production  $R_{uds}(s)$ , the continuum  $c\bar{c}$  production  $R_{(c)}(s)$ , and the bare  $\psi'$  resonance production (here and below, the  $\psi(3770)$  is denoted as  $\psi'$  for short), which is written as

$$R_{uds(c)+\psi'}^{th}(s) = R_{uds}(s) + R_{(c)}(s) + \frac{\sigma(e^+e^- \rightarrow \psi' \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} , \quad (3)$$

with  $R_{(c)}(s) = f_{(c)}p_{0,\pm}^3/E_{0,\pm}^3$  in BESIII's fit [4]. The  $\sigma(e^+e^- \rightarrow \psi' \rightarrow \text{hadrons})$  are the hadrons production cross section through the bare  $\psi'$  resonance in  $e^+e^-$  annihilation, and it could be written in terms of the form factor  $F_{\psi'}(s)$  in the following way:

$$\sigma(e^+e^- \rightarrow \psi' \rightarrow \text{hadrons}) = \frac{8\pi\alpha^2}{3s^{5/2}}[p_0^3(s) + p_{\pm}^3(s)]|F_{\psi'}(s)|^2 , \quad (4)$$

where besides the factor from phase space, the bare  $\psi'$  form factor  $F_{\psi'}(s)$  would be taken as the BW form:

$$F_{\psi'}(s) = \frac{g_{\psi'D\bar{D}}g_{\psi'\gamma}}{s - m_{\psi'}^2 + im_{\psi'}\Gamma_{\psi'}(s)} , \quad (5)$$

where  $g_{\psi'D\bar{D}}$  and  $g_{\psi'\gamma}$  are the coupling constants of the  $\psi'$  to the  $D\bar{D}$  and photon, respectively. Experiments indicate that the dominated decay channel of  $\psi'$  resonance is the  $D\bar{D}$ . Hence, we may use the energy dependent width

$$\Gamma_{\psi'}(s) = \Gamma_{D\bar{D}} + \Gamma_{nonD\bar{D}} = g_{\psi'D\bar{D}}^2 \frac{p_0^3(s) + p_{\pm}^3(s)}{6\pi s} + \Gamma_{nonD\bar{D}} , \quad (6)$$

or an improved parameterization of  $\Gamma_{D\bar{D}}$  in Eq. (2). However, as we addressed in Sec. 1, Eq. (5) is enough to describe the asymmetric line shape of the  $\psi(3770)$  state, but does not describe well the dip around 3.82 GeV. The main weakness of above treatment is the totally separation of the continuum and resonant  $D\bar{D}$  production in Eq. (3), but in fact, they are convoluted to each other. This is justified by the BESIII's fit results with the lower limit of  $f_{(c)} \sim 0$  within the uncertainties. Keeping this in mind, we correct the Eq. (3) as,

$$R_{uds(c)+\psi'}^{th}(s) = R_{uds}(s) + \frac{\sigma(e^+e^- \rightarrow (c\bar{c}) + \psi' \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} , \quad (7)$$

The  $\sigma(e^+e^- \rightarrow (c\bar{c}) + \psi' \rightarrow \text{hadrons})$  are the hadrons production cross section through the continuum  $c\bar{c}$  and the  $\psi'$  resonance in  $e^+e^-$  annihilation, which should be alike to Eq. (4) :

$$\sigma(e^+e^- \rightarrow (c\bar{c}) + \psi' \rightarrow \text{hadrons}) = \frac{8\pi\alpha^2}{3s^{5/2}}[p_0^3(s) + p_{\pm}^3(s)]|F_{(c)+\psi'}(s)|^2 , \quad (8)$$

Instead of Eq. (5), the Fano-type form factor including the interference between resonance and continuum background could be written as [17, 18, 26]

$$|F_{(c)+\psi'}(s)|^2 = |g_{\psi'D\bar{D}}g_{\psi'\gamma}F_{(c)}|^2 \frac{|q + \varepsilon|^2}{1 + \varepsilon^2} , \quad (9)$$

with  $\varepsilon = (-s + m_{\psi'}^2)/(m_{\psi'}\Gamma_{\psi'})$ . In the present context, the Fano line shape parameter  $q$  characterizes the relative transition strength into the  $\psi'$  state versus the  $D\bar{D}$  continuum and can be related to the electromagnetic transition probability of the  $\psi'$  state. It is an energy dependent variable in the original formula but regreded as a constant in the limited energy range of present interest. The factor  $F_{(c)}$  comes from the non-resonant background possibly associating with either the direct  $\gamma^* \rightarrow D\bar{D}$  transition or the nearby  $\psi(2S)$  or other charmonium states. Because the background contribution would be different in various channels, the line shapes of the  $\psi'$  would not be the same in other channels, e.g.  $\psi' \rightarrow p\bar{p}$  [28] and  $p\bar{p}\pi^0$  [29]. This is obviously true for other hadron states. However, here we do not dig into this issue and parameterize  $F_{(c)}$  as  $F_{(c)}(s) = 1/(s - m_{bg}^2 + im_{bg}\Gamma_{bg})$  for simplicity. It should be pointed out that the  $F_{(c)+\psi'}(s)$  could be parameterized in other format, e.g. the coupled-channel models [13, 14, 18], however at the price of more complex.

The measured  $R_{uds(c)+\psi'}^{ex}$  values versus c.m. energies are taken from BESIII's report [4], as shown in Fig. 1 with statistical error bars. We fit these  $R_{uds(c)+\psi'}^{ex}(s)$  values at each energy point to the theoretical formula described above using the least squares fitting method. The objective function of the least squares to be minimized in the fit is defined as

$$\chi^2 = \sum_{i=1}^{68} \left( \frac{R_{uds(c)+\psi'}^{ex}(s_i) - R_{uds(c)+\psi'}^{th}(s_i)}{\Delta R_{uds(c)+\psi'}^{ex}(s_i)} \right)^2, \quad (10)$$

where  $R_{uds(c)+\psi'}^{ex}(s_i)$  is the measured value with the statistical error  $\Delta R_{uds(c)+\psi'}^{ex}(s_i)$  at the c.m. energy  $s_i$ , and  $R_{uds(c)+\psi'}^{th}(s_i)$  is the corresponding theoretical value calculated by Eq. (7).

In the considered narrow energy range, the  $R_{uds}$  could be viewed as a constant, independent of the energy. The  $\Gamma_{nonD\bar{D}}$  in Eq. (6) tends to be in the range of 0 ~ 5 MeV with large uncertainty in various fitting strategies and we do not include it into the following fits. So we have seven free parameters ( $R_{uds}$ ,  $q$ ,  $m_{\psi'}$ ,  $g_{\psi'D\bar{D}}$ ,  $g_{\psi'\gamma}$ ,  $m_{bg}$  and  $\Gamma_{bg}$ ) in total. Because  $g_{\psi'\gamma}$  could be determined by the leptonic width  $\Gamma_{\psi'e^+e^-} = 4\pi\alpha^2 g_{\psi'\gamma}^2 / (3m_{\psi'}^3)$ , we perform two separate fits to the data. One of them (Fit i) is to fix  $g_{\psi'\gamma} = 0.2523$  by the  $\Gamma_{\psi'e^+e^-} = 0.262$  KeV in Particle Data Group [27], and the other is to let it being a free parameter (Fit ii). The curves in Fig. 1 show these fits, where the solid line is Fit i and the dashed line is Fit ii. The corresponding fitted parameters are shown in Table 1, in which the errors are only statistical ones. The achieving  $\chi^2/ndf = 1.38$  and 1.23 respectively for Fit i and ii are obviously smaller than BESIII' result  $\chi^2/ndf = 94/61 = 1.54$  [6]. Particularly, Fit ii gives a dip around 3.82 GeV, which describes the data perfectly well. It gives  $R_{uds} = 2.156 \pm 0.022$ , whose central value is in excellent agreement with the prediction of pQCD [2] and can directly be used to evaluate the strong coupling constant  $\alpha(s)$  at the energy scale of around 3 GeV. However, the errors of the  $R_{uds}$  are on the same level of BESIII' result, and it still has some room for the improvement of the fit quality. These are probably due to the big uncertainties of the data, whose systematic errors are around the same scale of the statistical errors and not included here yet [4]. As a reference, the  $\chi^2/ndf$  could be close to 1.0 within above formula in a similar fit to the data of  $e^+e^- \rightarrow D\bar{D}$  reactions [18].

	Fit i	Fit ii
$R_{uds}$	$2.165 \pm 0.024$	$2.156 \pm 0.022$
$q$	$1.58 \pm 0.31$	$-0.19 \pm 0.21$
$m_{\psi'} \text{ (MeV)}$	$3784.4 \pm 2.7$	$3816.0 \pm 13.9$
$g_{\psi'D\bar{D}}$	$14.0 \pm 0.8$	$14.1 \pm 3.4$
$g_{\psi'\gamma} \text{ (GeV}^2\text{)}$	0.2523 (fixed)	$0.417 \pm 0.048$
$m_{bg} \text{ (MeV)}$	$3753.6 \pm 4.6$	$3767.4 \pm 2.6$
$\Gamma_{bg} \text{ (MeV)}$	$37.9 \pm 3.2$	$41.9 \pm 6.0$
$\chi^2/ndf$	$85.52/62 = 1.38$	$74.94/61 = 1.23$

Table 1: Fitted parameters and achieving  $\chi^2/ndf$  in Fit i and Fit ii, see text for details.

The parameter  $q$  has big error in Fit ii. As can be seen in Eq. (9), its value largely rests on the position of dip in the line shape, which is however has large uncertainty. So it could deduce that the uncertainty of  $q$  in Fit ii comes from the coincidence of the fitted  $m_{\psi'}$  and the dip position. In addition, the sign of  $q$  varies in Fit i and ii, and this is caused by the fitted  $m_{\psi'}$  in these two fits are lying on the opposite sides of the dip position.

It is found that the fitted  $m_{\psi'}$  both in Fit i and ii are larger than the BW values in PDG [27], even considering their big uncertainties. Our obtained values should be treated as bare mass of the  $\psi'$  as argued in Ref. [17] and depend on the way of dealing with the background term  $F_{bg}$ . However, the corresponding dressed mass would be close to the PDG values and more sophisticated models are involved to extract its value [18]. The width  $\Gamma_{\psi'} \sim 29$  MeV at the nominal mass  $m_{\psi'} = 3.773$  GeV calculated with the obtained  $g_{\psi'D\bar{D}}$  is consistent with the BW values in PDG. The obtained  $g_{\psi'\gamma}$  in Fit ii is bigger than that of the PDG value and the corresponding leptonic width  $\Gamma_{\psi'e^+e^-}$  is around 2.7 times bigger than that of the PDG. This is possibly due to the rough treatment of the parameterization of the  $F_{bg}$ . In Eq. (9), the value of  $g_{\psi'\gamma}$  is directly relevant to the form of  $F_{bg}$ .

The physical interpretation of  $m_{bg}$  and  $\Gamma_{bg}$  would be difficult in present framework, though they are expected to be associated to the parameters of  $\psi(3686)$  state [11, 12, 13, 14, 15, 16]. However, it is impossible to accurately

determine the mass and width of the  $\psi(3686)$  by the present data. It should be stressed that the extracted  $R_{uds}$  is stable and reliable regardless of the uncertainties of these parameters, as long as the line shape of  $\psi'$  state is correctly reproduced.

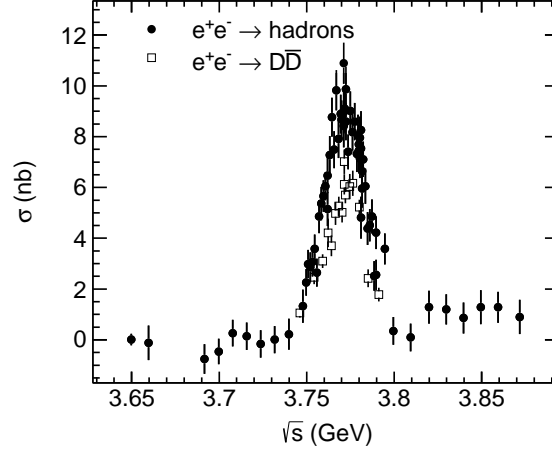


Figure 2: Cross sections of the  $e^+e^- \rightarrow (c\bar{c}) + \psi' \rightarrow \text{hadrons}$  reaction extracted from measured R ratios at different c.m. energies compared to that of the  $e^+e^- \rightarrow D\bar{D}$  [9].

Using the  $R_{uds}$  value extracted in Fit ii, we can obtain the cross sections of  $e^+e^- \rightarrow (c\bar{c}) + \psi' \rightarrow \text{hadrons}$  by Eq. (7), as shown in Fig. 2. The data of the  $e^+e^- \rightarrow D\bar{D}$  cross section from BESIII collaboration [5, 9, 22] are plotted in the same figure for comparison. The peak of  $D\bar{D}$  production cross section is obviously smaller and narrower than that of  $\text{hadrons}$ , which hints a non-zero  $\Gamma_{\text{non}D\bar{D}}$ . We calculate the cross section of  $\text{hadrons}$  production to be  $7.40 \pm 0.69$  (stat.) nb at  $\sqrt{s} = 3774.2$  MeV, which is bigger than the recent CLEO results  $\sigma(e^+e^- \rightarrow D\bar{D}) = 6.489 \pm 0.025 \pm 0.070$  at  $\sqrt{s} = 3774 \pm 1$  MeV. However, they are still consistent with each other when both the statistical and systematic uncertainties are taken into account. Thus, the non- $D\bar{D}$  decay ratio of the  $\psi'$  is waiting for more precise measurements.

### 3. Summary

In short summary, we have performed a renewed analysis of the measured  $R_{uds}$  value from BESIII collaboration by treating the anomalous line shape of the  $\psi(3770)$  resonance with a Fano-type formula. Our fitting results are better than those in a simple Breit-Wigner resonance with energy dependent width, mainly because of the improvement on the description of the dip structure at about 3.82 GeV. The  $R_{uds}$  value is determined to be  $2.156 \pm 0.022$  in the energy region between 3.650 and 3.872 GeV from the data of BESIII Collaboration. The central value is consistent with the pQCD calculation. We also reliably extract the cross sections of the  $e^+e^- \rightarrow \text{hadrons}$  without the continuum light hadron production, which would be beneficial to our understanding of the properties of the  $\psi(3770)$  state.

Our prescription of fitting the asymmetry line shape of states is not only useful for pinning down the controversial decay ratios of  $\psi(3770)$  state, but also meaningful for determining the R value in higher energy region where is often overlapped with other resonances. The proposed framework is easily extended to study other asymmetric line shapes of states and could be served as a simple fitting strategy to the experimental data.

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